

# **NUCLEAR LEVEL DENSITIES: DEPENDENCE ON PAIRING CORRELATIONS, ANGULAR MOMENTUM AND PARITY**

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# Motivation

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- a crucial ingredient for the calculation of scattering cross sections:

Fermi's golden rule: 
$$d\sigma(E) \propto \frac{2\pi}{\hbar} |\langle f | \hat{V} | i \rangle|^2 \rho(E) dE$$

- angular momentum distribution + parity dependence:
  - ▣ statistical nuclear reaction rates (thermal stellar reactions)
  - ▣ determination of NLD from measured proton or neutron resonances

# Calculating NLD

- Back-shifted Bethe formula (semi-empirical)

$$\rho_{BBF}(E_x) = \frac{\sqrt{\pi}}{12} a^{-\frac{1}{4}} (E_x - \Delta)^{-\frac{5}{4}} e^{2\sqrt{a(E_x - \Delta)}}$$

*H. Bethe, Phys. Rev. 50, 332 (1936); Rev. Mod. Phys. 9, 69 (1937).*

*T. von Egidy and D. Bucurescu, Phys. Rev. C 72, 044311 (2005).*

- HF+BCS (shell, pairing, deformation effects)

*P. Demetriou and S. Goriely, Nucl. Phys. A695, 95 (2001).*

- Shell Model Monte Carlo (pairing+quadrupole)

*S.E. Koonin, D.J. Dean and K. Langanke, Phys. Rep. 278, 1 (1997).*

*W.E. Ormand, Phys. Rev. C, 56, R1678 (1997).*

*H. Nakada and Y. Alhassid, Phys. Rev. Lett. 79, 2939 (1997).*

*Y. Alhassid, G. F. Bertsch, S. Liu, and H. Nakada, Phys. Rev. Lett. 84, 4313 (2000).*

$$\rho = (2\pi\beta^{-2}C)^{-1/2} e^S$$

# Angular momentum and parity dependence

- Spin cut-off model (uncorrelated randomly coupled spins)

$$\rho_J(E_x) = \rho(E_x) \frac{(2J+1)}{2\sqrt{2\pi}\sigma^3} e^{-\frac{J(J+1)}{2\sigma^2}}$$

*T. Ericson, Adv. Phys. 9, 425 (1960).*

- Parity projection in SMMC:

strong parity dependence in total NLD ( $^{56}\text{Fe}$ ,  $^{60}\text{Ni}$ ,  $^{68}\text{Zn}$ )

*Y. Alhassid, G. F. Bertsch, S. Liu, and H. Nakada, Phys. Rev. Lett. 84, 4313 (2000).*

Recent experiments: no parity dependence for J=2 states in  $^{90}\text{Zr}$  ( $E_x = 8-14$  MeV)  
 $^{58}\text{Ni}$

*Y. Kalmykov et al., Phys. Rev. Lett. 96, 012502 (2006).*

- Recently: projection on angular momentum in SMMC

Sign problem except for J=0!!!

*Y. Alhassid, S. Liu, and H. Nakada, nucl-th/0607062.*

# A pairing model

## SU(2) algebra

$$\begin{aligned}\hat{H} &= \sum_j 2\varepsilon_j \hat{S}_j^0 - \sum_{jj'} g_{jj'} \hat{S}_j^+ \hat{S}_{j'}^-, \\ \hat{S}_j^0 &= \frac{1}{2} \sum_{m>0} (\hat{a}_{jm}^\dagger \hat{a}_{jm} + \hat{a}_{j\bar{m}}^\dagger \hat{a}_{j\bar{m}} - 1), \\ \hat{S}_j^+ &= \sum_{m>0} \hat{a}_{jm}^\dagger \hat{a}_{j\bar{m}}^\dagger, \quad \hat{S}_j^- = \sum_{m>0} \hat{a}_{j\bar{m}} \hat{a}_{jm},\end{aligned}$$

- Formulate the pairing problem in **quasi-spin formalism**

*“Nuclear Superfluidity – Pairing in Finite systems”,  
D.M. Brink and R.A. Broglia, Cambridge University Press (2005).*

*$J$  is a good quantum number !!*

- simulate directly in quasi-spin basis (“spin chain”), with recently developed “canonical worm algorithm”

**No sign problem for all  $J$ ,  
and odd particle numbers !!**

$$\nu_j = \Omega_j - 2S_j$$

$$N_j = 2S_j^0 + \Omega_j$$

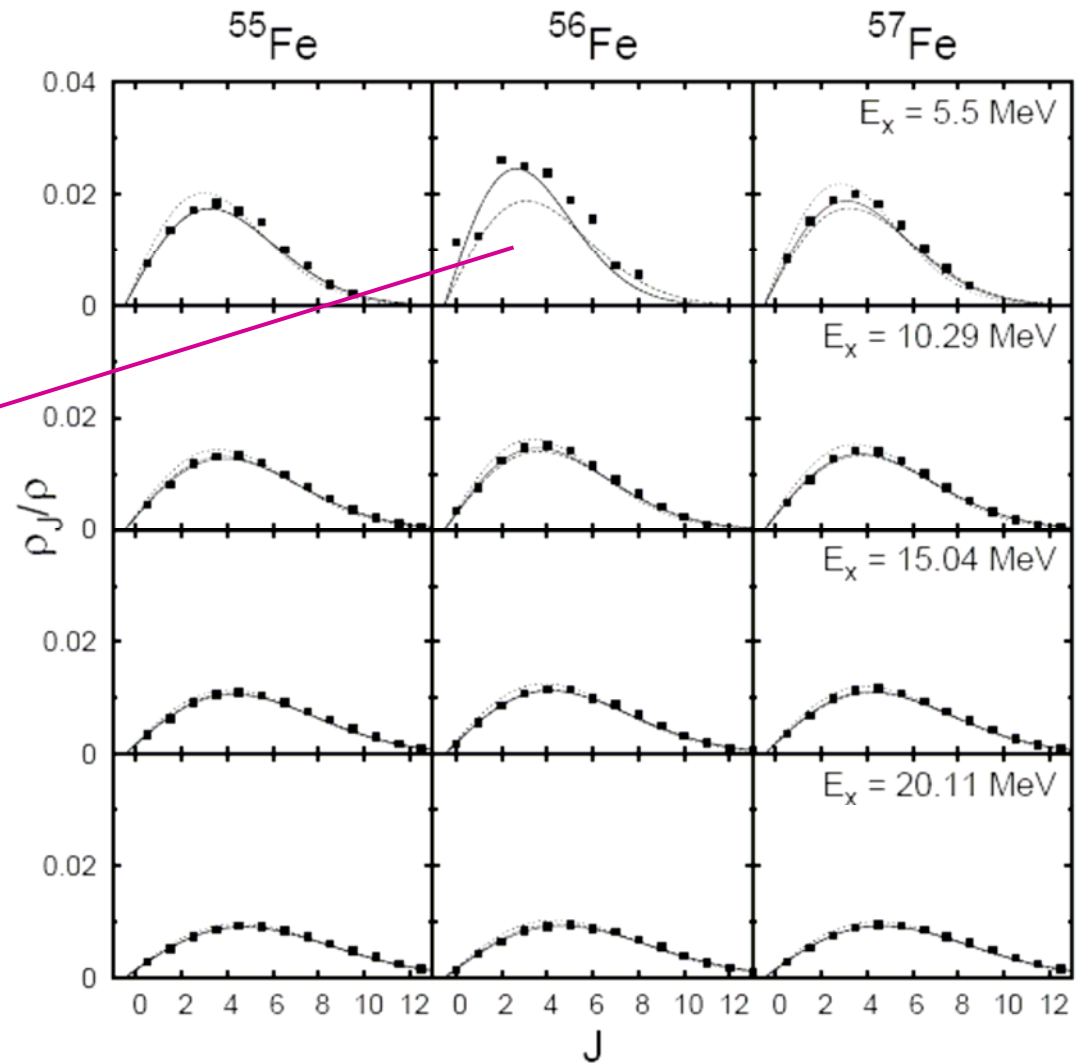
$$\Omega_j = j + 1/2$$

*S. Rombouts, K. Van Houcke and L. Pollet, Phys. Rev. Lett. 96, 180603 (2006).  
K. Van Houcke, S. Rombouts and L. Pollet, Phys. Rev. E 73, 056703 (2006).*

## Distribution in J

$$\rho_J(E_x) = \rho(E_x) \frac{(2J+1)}{2\sqrt{2\pi}\sigma^3} e^{-\frac{J(J+1)}{2\sigma^2}}$$

Signature of  
isovector pairing

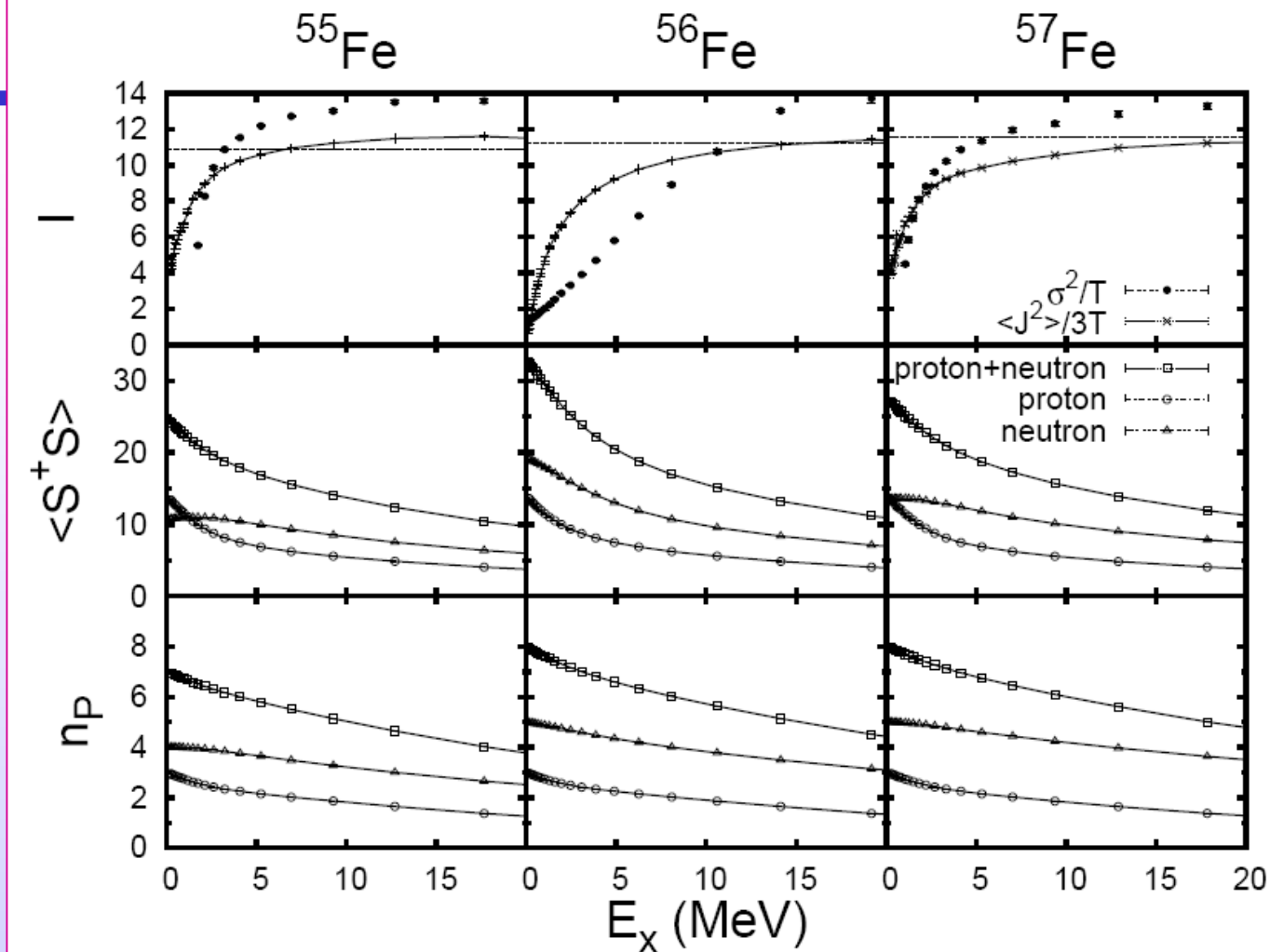


- effective moment of inertia:

$$I = \frac{\hbar^2}{T} \sigma^2$$

- spin cut-off parameter in semi-classical approach:

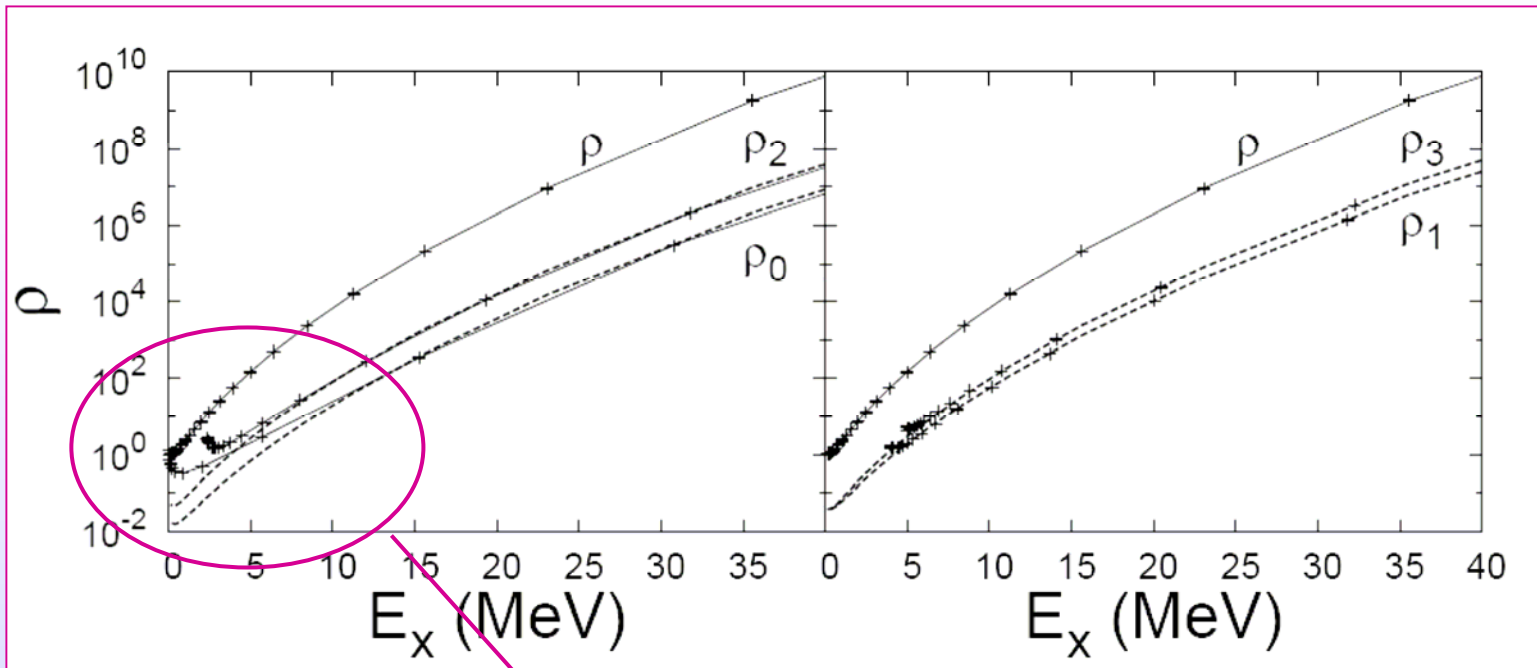
$$\sigma^2 = \frac{1}{3} \langle \hat{J}^2 \rangle,$$



Reduction of effective moment of inertia  
+ increasing pairing correlation energy

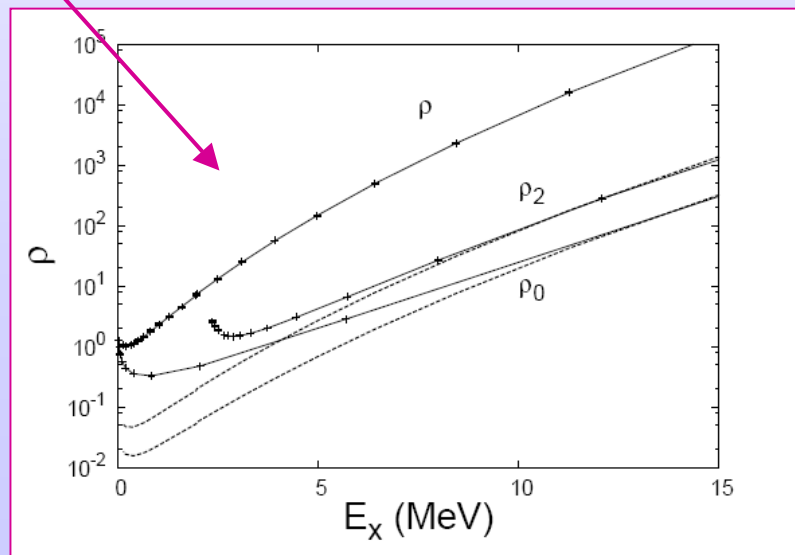
→ Nuclear  
Superfluidity

# Signatures of nuclear superfluidity in the NLD



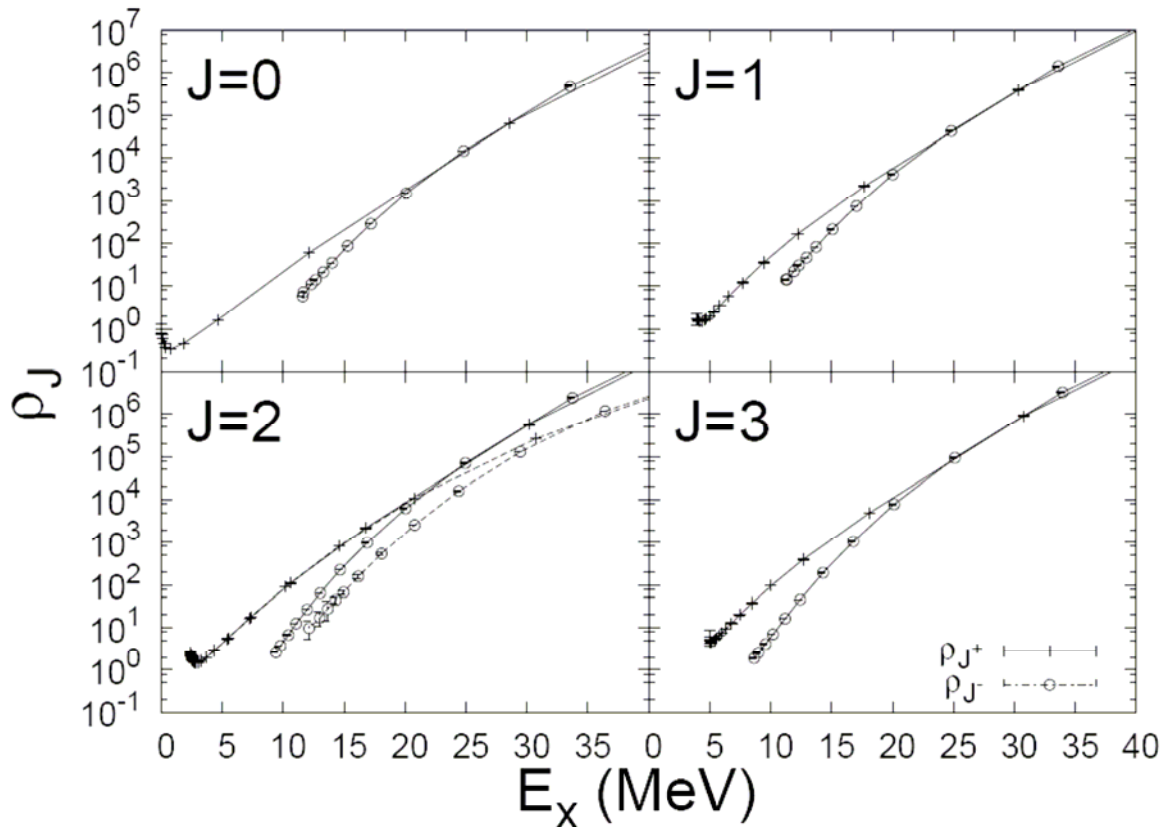
$^{56}\text{Fe}$

valence space:  
sd + pf + g<sub>9/2</sub> + sd





# Angular momentum + parity projection in $^{56}\text{Fe}$



strong parity dependence!!!

We consider sd + pf + g<sub>9/2</sub> + sd valence space

pf + g<sub>9/2</sub> valence space is too small for study of parity dependence!

# Conclusions and outlook

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- We solved general isovector pairing model at finite  $T$  via QMC
- Include angular momentum + parity projection without sign problem
- Signatures of pairing (nuclear superfluidity) can be found in the angular momentum distribution of the NLD
- Even projected on  $J$ , there is still a strong parity dependence in the NLD

*K. Van Houcke, S. Rombouts, K. Heyde and Y. Alhassid, in preparation.*

- Future: use deformed mean-field + pairing  
study the parity dependence